

Problem for the week of February 6, 2012

Let $M_2(\mathbb{R})$ be the vector space of all 2×2 real matrices. The standard basis for $M_2(\mathbb{R})$ is given as follows:

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Define the linear transformation

$$T(X) = XA - AX$$

where $X \in M_2(\mathbb{R})$, and $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

- (a) Find the matrix representation of T under the basis E_i , $i = 1, 2, 3, 4$.
- (b) Find a basis for the range of T .
- (c) Find a basis for the kernel of T .