Linear Algebra **Quiz 1**

4/6/2005

1. (40%) Suppose the complete solution to the equation

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \text{ is } \mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

   (a) (14%) What is the matrix A?

   (b) (10%) Find the reduced row echelon form of A.

   (c) (10%) Find a basis for the row space of A.

   (d) (6%) What is the rank of A?

2. (30%) Suppose A and B are m by n matrices.

   (a) (7%) True or false: If $A\mathbf{x}=\mathbf{b}$ has exactly one solution for some right sides $\mathbf{b}$, then dim $C(A)=n$. Give a reason or counterexample.

   (b) (7%) If $A\mathbf{x}=\mathbf{b}$ has no solution for some right sides $\mathbf{b}$ and infinitely many solutions for some other right sides $\mathbf{b}$, what are all equations or inequalities that must hold between the numbers m, n, and r? Note that $r = \text{rank}(A)$.

   (c) (8%) True or false: If A and B have the identical four subspaces, i.e., $C(A)=C(B)$, $N(A)=N(B)$, $C(A^T)=C(B^T)$, and $N(A^T)=N(B^T)$, then $A=B$. Given a reason or counterexample.

   (d) (8%) True or false: $\text{rref}(A)^T=\text{rref}(A^T)$. Give a reason or counterexample. Note that rref(A) stands for the reduced row echelon form of A.

3. (30%) Suppose

$$A = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 2 & a \end{bmatrix}.$$

   (a) (10%) If possible, find the number a that makes rank(A)=3, or give a reason why such an a does not exist.

   (b) (10%) If a=10, find a basis for the nullspace of A.

   (c) (10%) Find the number a so that the column space of A spanned by

$$\begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}.$$