

Linear Algebra**Quiz 1**

4/10/2007

1. (22%) Suppose $A = \begin{bmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ 2 & b & 0 \end{bmatrix}$.

(a) (8%) Find the second column of A^{-1} (if it exists) for A .

Hint: To find the second column of A^{-1} , what system $A\mathbf{x}=\mathbf{b}$ would you solve?

(b) (8%) For each a and b , find the rank of A and say why. Separate the discussions for different cases of a and b .

(c) (6%) If $a=2$ and $b=3$, find the matrix B such that $AB = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$.

2. (24%) Suppose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are a basis for a subspace of R^4 , and they are column vectors of A .

(a) (6%) How do you know that $A\mathbf{x}=\mathbf{0}$ has only the solution $\mathbf{x}=\mathbf{0}$?

(b) (6%) How do you know that $A^T\mathbf{y}=\mathbf{0}$ has a solution $\mathbf{y}\neq\mathbf{0}$?

(c) (6%) Is it true that $A\mathbf{x}=\mathbf{b}$ has exactly one solution for any right side \mathbf{b} and why?

(d) (6%) Is it true that A has linearly independent rows and why?

3. (20%)

(a) (10%) If A is invertible, explain why AA^T is also invertible.

(b) (10%) If $A^2=0$, the zero matrix, explain why A is not invertible.

4. (34%) Suppose after elimination on a matrix A we obtain its reduced row

echelon form as follows: $R = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(a) (8%) Find the null space matrix of A .

(b) (8%) What is the null space of A^T ? Find a basis for it.

(c) (10%) Find a complete solution to $R\mathbf{x}=\mathbf{c}$ where $\mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(d) (8%) What is the rank of 3-by-8 block matrix $[A \ A]$?