Linear Algebra **Quiz 1**

4/11/2008

1. (30%) Suppose

$$A=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & 3 & 4\\ 2 & 1 & 0 & -1\end{bmatrix}$$

(a) (7%) What is the rank of A?

(b) (7%) Find a basis for the column space of A.

(c) (7%) Find a basis for the nullspace of A^T.

(d) (9%) Find a 1 by 3 matrix B such that the column space of A equals the nullspace of B, $C(A)=N(B)$. (Hint: how can you tell whether $\mathbf{b}$ is in the column space of A?)

2. (40%) Suppose the general solution to the equation $A\mathbf{x}=\mathbf{b}=\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}$ is

$$\mathbf{x}=\begin{bmatrix}2\\ 3\\ 6\\ 1\end{bmatrix}+\alpha\begin{bmatrix}-2\\ 1\\ 0\\ 0\end{bmatrix}+\beta\begin{bmatrix}-3\\ 0\\ 1\\ 0\end{bmatrix}.$$

(a) (7%) What is the reduced row echelon form of A?

(b) (7%) What is the rank of the augmented matrix $\begin{bmatrix}A|\mathbf{b}\end{bmatrix}$?

(c) (8%) Is the matrix A^TA invertible? Give a reason.

(d) (8%) Are you sure that the particular solution $\begin{bmatrix}2\\ 3\\ 6\\ 1\end{bmatrix}$ is in the row space of A? Yes or no. Give a reason.

(e) (10%) Suppose $B=A\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$. Is $B\mathbf{x}=\mathbf{b}$ solvable? If your answer is yes, show the general solution; otherwise, give a reason.

3. (30%) True or false. Give a reason or counterexample.

Suppose A and B are 4 by 3 matrices.

(a) (10%) If $A\mathbf{x}=\mathbf{0}$ only has the trivial solution $\mathbf{x}=\mathbf{0}$, then A has a left inverse, i.e., there exists a 3 by 4 matrix C such that $CA = I$.

(b) (10%) If A and B have the same column space $C(A)=C(B)$ and the same nullspace $N(A)=N(B)$, then $A=B$.

(c) (10%) If A and B have the same reduced row echelon form, then they have the identical nullspaces and left nullspaces, i.e., $N(A)=N(B)$ and $N(A^T)=N(B^T)$.