Linear Algebra **Quiz 2**

5/11/2005

You must show your work to receive credit.

1. (40%) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 3 \\ 2 & 4 & 2 & 4 \end{bmatrix}$.

   (a) (11%) Find the solution to $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ that is closest to $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

   (b) (10%) Give an orthonormal basis for the nullspace of A.

   (c) (8%) Find the projection matrix P projecting onto the row space of A.

   (d) (6%) Find the vector in the row space of A that is closest to $\mathbf{b}$.

   (e) (5%) Explain why the determinant of every projection matrix is either 0 or 1.

2. (30%) Answer the following questions.

   (a) (7%) Suppose Q is a 4 by 3 matrix with orthonormal columns $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$. Is it true that $QQ^T=I$, where I is the 4 by 4 identity matrix? Justify your answer.

   (b) (8%) True or false: If $A^TAx=0$ then $A\mathbf{x}=\mathbf{0}$.

   (c) (7%) True or false: A^TA and A have the same column space.

   (d) (8%) True or false: Suppose A and B are n by n matrices. If $A\mathbf{x}=\mathbf{b}$ has exactly one solution for every right side $\mathbf{b}$, and $B\mathbf{x}=\mathbf{0}$ has nonzero solutions, then $C=AB$ must be singular. Give a reason or counterexample.

3. (30%) Answer the following questions.

   (a) (15%) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Use the Cramer's (or cofactor) rule to find A^{-1}, where $A^{-1} = \dfrac{C^T}{\det A}$ and C is the cofactor matrix of A.

   (b) (7%) Is it true that the inverse of a triangular matrix is always triangular? Justify you answer.

   (c) (8%) Suppose A is n by n. Find det C.