**Linear Algebra** **Quiz 2**

5/17/2007

**You must show your work to receive credit.**

1. (34%) Let $A=\begin{bmatrix} 2 & 6 & 2 \\ 1 & 3 & 1 \end{bmatrix}$.

   (a) (10%) Find the 3 by 3 projection matrix $P$ onto the row space of $A$ (after closely examining the matrix).

   (b) (6%) What is the closest vector in the row space of $A$ to the vector $\mathbf{b}=(0,1,0)^T$.

   (c) (6%) Explain why $AP=A$. (What does the projection $P$ do?).

   (d) (12%) Find an orthonormal basis for the subspace of all vectors orthogonal to the row space of $A$.

2. (22%) Suppose $Q$ is an $m$ by $n$ matrix with orthonormal columns.

   (a) (6%) What is the rank of $Q$?

   (b) (8%) Suppose $P$ is the projection matrix onto the column space of $Q$ in (a). What is the rank of $P$?

   (c) (8%) True or false: rank($QQ^T$)=rank($Q^TQ$) (I would use the relation rank($A^TA$)=rank($A$)).

3. (44%) Answer the following questions.

   (a) (10%) Using the Cramer's rule, find $b_1$ such that $x_3=0$ for the solution of

$$A\mathbf{x}=\begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} b_1 \\ 1 \\ 3 \end{bmatrix}.$$

   (b) (10%) Compute the determinant of the cofactor matrix of $A$ in (a). (Don't do a lot of calculations…please.)

   (c) (14%) Let **u**, **v**, **w** be linear independent vectors in $R^3$. How is the matrix $A$ with columns **u**, **v**, **w** related to the matrix $B$ with columns **u**−**v**, **u**+**v**, **u**+2**v**−**w**? Show that matrix $B$ has linear independent columns.

   (d) (10%) Using rules for the determinant (so do not do it with any of the formulas for computing determinant), show the steps and rules that lead to

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}=0$$