Linear Algebra **Exam 1**

3/31/2010

1. (32%) Suppose $A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 2 \\ 0 & 0 & k & k \\ 0 & 0 & k & k^2 \end{bmatrix}$.

   (a) (12%) Find all possible values of k so that rank(A) = 3.

   (b) (10%) If rank(A) = 2, find a basis for the column space of A.

   (c) (10%) Do A and U have the same nullspace? Why or why not.

2. (36%) Suppose A is reduced by elementary row operations to $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) (10%) Is $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ solvable or is it impossible to tell?

   (b) (12%) Is $A^T\mathbf{y} = \begin{bmatrix} 1 \\ 3 \\ -1 \\ -2 \end{bmatrix}$ solvable or is it impossible to tell?

   (c) (14%) Let $\mathbf{b}$ be the sum of the columns of A. Find the general solution to the system $A\mathbf{x} = \mathbf{b}$.

3. (32%) True or false. Give a reason or a counterexample.
   Suppose A and B are 4 by 3 matrices.

   (a) (8%) If A and B have the same reduced row echelon form, then rank(A) = rank(B).

   (b) (8%) If $AB^T = 0$, then the rows of A are in the nullspace of B.

   (c) (8%) If A and B have the same column space, then they have the same nullspace.

   (d) (8%) If $A^TA = I_3$, then the columns of A are linearly independent.