Linear Algebra **Exam 1**

4/10/2012

1. (40%)

Suppose $PA = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ b & c & 1 & 0 \\ d & e & f & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & -2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$

(a) (10%) Is $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ in the nullspace of A or is it impossible to tell?

(b) (10%) Do A and U have the same column space? Why or why not.

(c) (10%) Let $\mathbf{a}_i$ be the ith column of A. Find c_1 and c_2 so that $\mathbf{a}_3 = c_1\mathbf{a}_1 + c_2\mathbf{a}_2$.

(d) (10%) Find P^{99}.

2. (20%) Let A be an n by n invertible matrix.

(a) (10%) What is the reduced row echelon form of $\begin{bmatrix} A & I \\ I & 0 \end{bmatrix}$?

(b) (10%) Find the inverse of the block matrix $\begin{bmatrix} I_n & B \\ 0 & A \end{bmatrix}$.

3. (40%) True or false. Give a reason or counterexample. Suppose A and B are 3 by 4 real matrices.

(a) (8%) It is impossible that the column space of A has basis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and the nullspace of A has basis $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

(b) (8%) For every A, there exists $\mathbf{x} \neq \mathbf{0}$ such that $A\mathbf{x} = \mathbf{0}$.

(c) (8%) If A and B have the same column space, then they have the same reduced row echelon form.

(d) (8%) It is impossible to find A and B such that $AB^T = I_3$.

(e) (8%) If $A^T A = 0$, then $A = 0$.