Linear Algebra **Exam 1**

4/2/2013

1. (50%) Suppose $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 & 4 & 2 & 5 \\ 0 & 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.
   - (a) (10%) What is the third row of A?
   - (b) (10%) Is $\begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$ in the row space of A or is it impossible to tell?
   - (c) (10%) Find a basis for the nullspace of A.
   - (d) (10%) Find the complete solution of $A^T\mathbf{y} = \mathbf{0}$.
   - (e) (10%) Is $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$ consistent (solvable)? Why or why not?
2. (18%) Answer the following questions.
   - (a) (9%) If A, B and A+B are invertible, show that $A(A+B)^{-1}B = B(A+B)^{-1}A$.
   - (b) (9%) Let A be an invertible matrix. What is the inverse of $\begin{bmatrix} A & 0 \\ B & A \end{bmatrix}$?
3. (32%) True or false. Give a reason or counterexample.
   - (a) (8%) It is impossible to find a 4 by 3 matrix A so that $A\mathbf{x} = \mathbf{b}$ has at most one solution for every $\mathbf{b}$.
   - (b) (8%) It is impossible to find a 3 by 4 matrix A so that A^TA is invertible.
   - (c) (8%) If A^T and B^T have the same reduced row echelon form, then A and B have the same column space.
   - (d) (8%) It is impossible that the row space of A has basis $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and the nullspace of A has basis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (Hint: What is the reduced row echelon form of A?)