Linear Algebra **Exam 1**

4/9/2015

1. (52%) Suppose $E[A\,|\,\mathbf{b}] = \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & b_2 - 4b_1 \\ 0 & 0 & 0 & b_3 - 2b_2 + b_1 \end{bmatrix}$, where E is a 3 by 3 invertible matrix.

   (a) (6%) What is the rank of A?

   (b) (8%) Is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in the row space of A? Why or why not.

   (c) (8%) Is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in the column space of A? Why or why not.

   (d) (10%) What is the matrix E?

   (e) (10%) Find the complete solution to $A\mathbf{x} = \mathbf{0}$.

   (f) (10%) Find a basis for the left nullspace of A.

2. (16%) Find a basis for each of the following subspaces in $\mathbb{R}^4$:

   (a) (8%) All vectors whose components are equal.

   (b) (8%) All vectors whose components add to zero.

3. (32%) True or false. Give a reason or counterexample. Suppose A is an m by n real matrix of rank r, and $\mathbf{b} \in \mathbb{R}^m$..

   (a) (8%) If the rank of $[A\,|\,\mathbf{b}]$ is equal to r, then $A\mathbf{x} = \mathbf{b}$ is solvable (consistent).

   (b) (8%) If AA^T is invertible, then $m \le n$.

   (c) (8%) If there are some $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has no solution, then $m \ge n$.

   (d) (8%) If $A\mathbf{x} = \mathbf{b}$ has infinite solutions for every $\mathbf{b}$, then $m < n$.