Linear Algebra **Exam 1**

3/31/2016

1. (56%) Suppose $E[A\,|\,I_3]=\left[\begin{array}{cccc|ccc}1&2&-1&1&3&2&4\\0&0&0&1&2&1&3\\0&0&0&0&1&1&0\end{array}\right]$, where E is a 3 by 3 invertible matrix.
   (a) (6%) What is the dimension of the column space of A?
   (b) (6%) What is the dimension of the left nullspace of A?
   (c) (6%) Is $\begin{bmatrix}0\\0\\1\end{bmatrix}$ in the column space of A? Explain.
   (d) (6%) Is $\begin{bmatrix}1\\2\\-1\\3\end{bmatrix}$ in the row of A? Explain.
   (e) (6%) Let $\mathbf{a}_j$ be the jth column of A. Find c so that $c\mathbf{a}_2+\mathbf{a}_3=\mathbf{0}$.
   (f) (8%) For which values of b_1, b_2 and b_3 is $A\mathbf{x}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$ consistent?
   (g) (8%) Find a basis for the nullspace of A.
   (h) (10%) Find the first column of A.

2. (44%) True or false. Give a reason or counterexample.
   (a) (7%) Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be nonzero vectors in $\mathbb{R}^2$. If $\mathbb{R}^2=\operatorname{span}\{\mathbf{u},\mathbf{v},\mathbf{w}\}$, then $\mathbf{w}$ must be a linear combination of $\mathbf{u}$ and $\mathbf{v}$.
   (b) (7%) If A is a 3×4 matrix of rank(A)=2, there are six types of matrices in reduced row echelon form.
   (c) (7%) If AB=0 for two 2×2 matrices A and B, then BA must be the zero matrix as well.
   (d) (7%) Suppose A is am $m\times n$ matrix. If $A\mathbf{x}=\mathbf{b}$ has at least one solution for every $\mathbf{b}$, then there exists a matrix B such that $BA=I_n$.
   (e) (8%) If A is an $n\times n$ nonsingular matrix and B is an $n\times p$ matrix, then $N(B)$=$N(AB)$.
   (f) (8%) There exists a 3×3 matrix A such that $C(A)$=$N(A)$.