Linear Algebra **Exam 2**

5/17/2012

You must show your work to receive credit.

1. (36%) . Answer the following questions.

   (a) (10%) Let $T:\mathbb{R}^3 \to \mathbf{P}^2$ be a linear transformation. If $T\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\} = 1+t+t^2$, $T\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}\right\} = 1+t^2$, $T\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}\right\} = 3+t+3t^2$, find the kernel of *T*, i.e., the set of **x** in $\mathbb{R}^3$ such that $T(\mathbf{x}) = 0$.

   (b) (10%) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. The matrix representation of *T* with respect to the basis $B = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$ is $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$. Find the matrix representation of *T* with respect to the basis $C = \left\{\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$.

   (c) (8%) Are the following three polynomials linearly independent?
   $$1-t+t^3,\ t-2t^2,\ 2-t-2t^2+t^3.$$

   (d) (8%) Suppose $T:\mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with $\text{rank}(T) = r$. If *T* is one-to-one, but does not map onto $\mathbb{R}^m$, what is the relationship between *r*, *m* and *n*?

2. (38%) Let $A = \begin{bmatrix}1 & 1 & 2\\a & b & c\end{bmatrix}$ with $\text{rank}(A) = 1$. Let *P* be the 3 by 3 orthogonal projection matrix onto the nullspace of *A*.

   (a) (10%) Find an orthonormal basis for the nullspace of *A*.

   (b) (10%) Find *P*.

   (c) (10%) What is the closest vector in the row space *A* to the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$?

   (d) (8%) Is it true that $AP = 0$? Yes or no. Briefly explain why.

3. (26%) Answer the following questions.

(a) (8%)

$$\begin{vmatrix} 0 & 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 2 & 3 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & -1 & 0 & 1 \\ -4 & -3 & -2 & -1 & 0 \end{vmatrix} = ?$$

(b) (10%) Find the area of the parallelogram with corners (0,0,0), (1,1,1), (0,1,2), (1,2,3).

(c) (8%) Suppose A is an n by n nonsingular matrix, and $A = QR$, where Q is an n by n orthogonal matrix, and R is an n by n upper triangular matrix. Is it true that $\det A = \det R$? Yes or no. Give a reason or counterexample.