Linear Algebra **Exam 2**

5/14/2013

1. (38%) Let $A=\begin{bmatrix}1 & 1\\ -1 & 1\\ 1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$. Examine the matrices carefully before you answer the following questions.

   (a) (10%) Find the least-squares solution of $A\mathbf{x}=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$.

   (b) (10%) Let **z** be a 4-dimensional vector. Find all possible **z**'s so that $\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$ is the closest vector in the column space of A to **z**.

   (c) (10%) What is the orthogonal projection matrix onto the nullspace of A^T?

   (d) (8%) Let M be the orthogonal projection matrix onto the nullspace of AA^T. Is it true that MA=0? Why or why not.

2. (36%) Answer the following questions.

   (a) (10%) Let $T(x,y)=\begin{bmatrix}4x-2y\\ -5x+3y\end{bmatrix}$. Find the area of $T(S)$, where $S=\{(x,y)\mid -1\le x\le 1,\ 2\le y\le 3\}$.

   (b) (10%) Let $T:\mathbb{R}^2\to\mathbb{R}^2$ be a linear transformation and $\beta=\left\{\begin{bmatrix}1\\1\end{bmatrix},\ \begin{bmatrix}1\\0\end{bmatrix}\right\}$ be a basis for $\mathbb{R}^2$. If $A=\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$ is the matrix representation of T with respect to β, determine $T(2,5)$.

   (c) (8%) Let $p(t)$, $q(t)$ and $r(t)$ be linearly independent polynomials in $\mathbf{P}^2$. Is the set $\{p(t)+q(t),\ q(t)+r(t),\ p(t)+r(t)\}$ linearly independent? Why or why not.

   (d) (8%) Suppose A is a 6×3 matrix and B is a 3×5 matrix. If A is one-to-one and B maps $\mathbb{R}^5$ onto $\mathbb{R}^3$, find all possible values of rank(AB).

3. (26%) Answer the following questions.

(a) (10%) Find the determinant of $A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 & 3 \\ 0 & 0 & 5 & 3 & 5 \\ 5 & 3 & 9 & 8 & 4 \\ 8 & 5 & 4 & 6 & 3 \end{bmatrix}$.

(b) (8%) If all entries of A and A^{-1} are integers, prove that $\det A = 1$ or -1.

(c) (8%) Let A be an n by n matrix and C be the cofactor matrix of A. If A is singular, how do you know that C is also singular?